Miami Dade College.
MAC1105.
https://mystatclass.com/mac1105w.html

How to solve the two main types of absolute value inequalities.

The key to solving absolute value inequalities is to remember what absolute value means**: distance from zero**. Thinking about distance will help you understand why we use two different approaches.

Let's consider a positive number, **c.**

Type 1: Less Than ("And" Statements)
This type looks like $|A| < c \quad or \quad |A| \leq c.$

The Rationale: If the absolute value of something is less than c, it means its distance from zero is less than c. This means the value must be *trapped* or *sandwiched* between $-c$ and c. This is an **and** situation because the value must be simultaneously greater than $-c$ and less than c.

Therefore, if $|A| < c$, then we rewrite it as a single compound inequality:

$-c < A < c$

Then, solve for the variable.

<u>Example</u>: Solve $|2x - 1| < 5.$

1. Set up the sandwich inequality: Since it's a "less than" problem, we trap the expression between $-5 \; and \; 5.$

$-5 < 2x - 1 < 5$ which yields: $-2 < x < 3$ Interval Notation: $(-2, 3)$

Type 2: Greater Than (“Or” Statements)

This type looks like $|A| > c \quad \text{or} \quad |A| \geq c.$

The Rationale: If the absolute value of something is greater than c, it means its distance from zero is more than c. This can happen in two ways: the value is far to the right (greater than c or it's far to the left (less than $-c$). This is an "or" situation.

The Rule: If $|A| > c$, then we rewrite it as two separate inequalities joined by "or":

$A < -c \quad \text{or} \quad A > c$

Then, solve each inequality separately.

<u>Example</u>: Solve $|3x + 2| \geq 4.$

$3x + 2 \geq 4 \quad \text{or} \quad 3x + 2 \leq -4$

Solve each case:
Case 1: $3x + 2 \geq 4$ solution: $x \geq \frac{2}{3}$
Case 2: $3x + 2 \leq -4$ solution: $x \leq -2$